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# Post buckling control of damaged composite plates using piezoelectric patches

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Abstract: This work presents an efficient technique to control the post buckling characteristics of a damaged composite plate using piezo patches. This paper discusses about the usage of piezoelectric fibre composite patches (PFCP) for enhancing the post buckling performance and thereby reducing the effects of internal flaws. A finite element formulation based on Inverse Hyperbolic Shear Deformation Theory (IHSDT) for handling structural analysis of a smart composite plate is used in the present work. In addition to the best performance, reduction in weight of piezoelectric material is obtained as we employ a segmented piezo patch to overcome the degradation in post buckling strength due to damage in a composite plate, which indeed addresses the design issues.

Keywords: Piezoelectric fibre composites; Composite plate; Anisotropic damage; Finite element analysis.

## 1. INTRODUCTION

Analysis of piezo attached composite structures is a very attractive and demanding area of research. Understanding the superiorities of PFC material over existing actuators, PFCs became a significant focus of a number of researchers. Several benefits of Active Fibre Composites (AFCs) over monolithic ceramic actuators are listed in the comprehensive study by Bent (1997). Azzouz (2001) presented the finite element modeling of AFC. Williams et al. (2002) and Bellolia et al. (2004) also discussed the manufacturing processes, intended applications and relative benefits of different types of active fibre composite actuators. Certain drawbacks may occur when we use PFC as a layer in application, say, when composite structure experiences large deformation and/or the surface of the composite structure is geometrically unconformable. In those conditions fibres may break (because they are thin, brittle and continuous piezoelectric). Consequently it will affect the actuation-ability of the actuator. Usage of segmented piezoelectric fibre composite patch (PFCP) at decided location can overcome above problems. The chosen displacement field for present analysis is on the basis of Inverse Hyperbolic Shear Deformation Theory (IHSDT) which was proposed by Grover et al. (2013). They proposed new nonpolynomial shear deformation theories and implemented for structural responses of laminated composite and sandwich plates. They concluded that it shows enhanced performance similar to all prevalent higher order shear deformation theories comprising shear strain function. Sreehari and Maiti (2015) have done post buckling analysis under thermal environment using finite element method. Biswas et al. (2011) have extensively investigated the static and dynamic characteristics of damaged composite panels. They used an anisotropic damage formulation to parametrically model the damage.

Structures should be able to survive maximum probable forces acting on them and overcome the effects of minor damages arising in them. We can use smart materials along with structural components to make them survive more forces than what they are expected to. In the present investigation we use PFCPs to strengthen damaged composite structures, thereby controlling the post buckling deformations.

## 2. MATHEMATICAL FORMULATION

A laminated composite plate is considered and a finite element formulation is developed for present analysis of laminated composite plates with damage. In the present work, the displacement field taken for structural analysis of the laminated composite plate with attached piezoelectric material is on the basis of IHSDT. The attachment of piezoelectric patches causes coupling effect and results in both electrical and mechanical excitation. It can be stated in terms of stresses and electrical displacements by the direct as well as converse piezoelectric effect. The piezoelectric constant matrix for piezo fibre reinforced composite are used in the formulation. The governing equation for the laminated composite plate with bonded PFC is derived. The governing dynamic equation of the piezoelectric attached laminated composite plate for bending control analysis can be modified and can be represented globally as:

$$[K_{uu} + K_c]\{u\} = \{F_1\} - [K_{u\phi}]\{\phi\} \Rightarrow [K_{net}]\{u\} = \{F_1\} - [K_{u\phi}]\{\phi\} \quad (1)$$

Here $[K_{uu}]$, $[K_C]$, $[K_{u\Phi}]$ after assembling represents the generalized global stiffness matrices corresponding to mechanical degrees of freedom, additional constraints, and electromechanical coupling. Eq. (1) shows the prebuckling equilibrium. In the next step geometric stiffness matrix $[K_G]$ associated with the membrane forces is computed (Zienkiewicz, 1971). In the current study, von Karman type of nonlinearity is used. Authors had explained the buckling of composite structures earlier (Sreehari and Maiti, 2015). The critical buckling load is calculated by solving the linear eigenvalue problem:

$$[K_{net} + \lambda K_G]\{u\} = 0 \quad (2)$$

While investigating the behaviour in thermal environment, the stress-strain relations are modified incorporating the thermal coefficients (Sreehari and Maiti, 2015). The lowest eigenvalue is the critical buckling load. After buckling, the nonlinear stiffness matrix $[K_{nl}]$ is incorporated (Zienkiewicz, 1971) as,

$$([K_{net}] + [K_{nl}] + \lambda [K_G])\{u\} = \{0\} \quad (3)$$

Above equation is solved to trace the post buckling path. A numerical technique, modified Newton Raphson iterative scheme, is applied for the solution of post buckling equation.

The anisotropic damage is parametrically introduced into the mathematical formulation by using a parameter, which really signifies the reduction in the elastic property in the damage zone. This parameter is essentially a representation of reduction in effective area and is given by

$$\Gamma_i = \frac{A_i - A_i^*}{A_i} \quad (4)$$

where $A_i^*$ is the effective area (with unit normal) after damage and $i$ denotes the three orthogonal directions. For a thin plate, only $\Gamma_1$ and $\Gamma_2$ need to be considered. $\Gamma_1$ indicates the damage in the direction of the fibre while $\Gamma_2$ indicates orthogonal damage (in same plane). The effects of a region of damage are introduced by the usage of an idealized model having a lessening in the elastic property in the damage zone. This scheme which parametrically models damage in any anisotropic material was used recently (Sreehari et al., 2016) for finding the effects of damage in a smart plate. A complete Gauss quadrature rule (3 × 3) is used in finding element bending stiffness matrix and load vectors. But reduced integration (2 × 2) is used in finding element transverse shear stiffness matrix to avoid shear locking phenomenon in thin plates. The details of formulation are omitted in this abstract for the sake of brevity.

## 3. RESULTS AND DISCUSSION

Computer programming has been developed for the analysis of a damaged composite plate based on the FEM formulation discussed in the previous section. A $C^0$-continuous 8 noded iso-parametric element was employed for discretization of the laminated plate. Convergence study is conducted in the beginning of this analysis.

Some validation studies are performed after convergence study. The validation of the present mathematical formulation using IHSDT and Matlab programming is performed for a piezolaminated plate. The objective in this initial example is to find out the nondimensional central deflection of undamaged smart plate and to compare the results with existing results. The plate is discretized using a (10 × 10) mesh. Piezolayers is used on top of the substrate material. The smart composite plate is stacked in order as (P/0/90/0). The dimensions of laminated composite square plate are taken as (10 cm × 10 cm). The analysis are conducted for thin ($a/h$ = 100) composite plates. In the present analysis, the deflection of the center point is evaluated using IHSDT at 100 V potential on the PFC layer. All edges of composite plate are considered to be simply supported. A transverse sinusoidal load of intensity 40 $Nm^{-2}$ is applied on the plate. The material properties are taken as per Shiyekar and Kant (2011). It is observed from Table 1 that the present results matches well with those of the published results. The percentage variations of results with those of exact solution are also provided inorder to show the performance of present formulation using IHSDT. The present results are in good agreement with the exact solution (Mallik and Ray, 2004) than those of other available higher order results.

Table 1. Validation of nondimensional central deflection.

| Source | Nondimensional central deflection of smart plate | Error % |
|---|---|---|
| Present (IHSDT) | 0.789 | 0.2 |
| Shiyekar and Kant (2011) (higher order theory) | 0.775 | 1.5 |
| Ray and Mallik (2004) (first order theory) | 0.763 | 3.1 |
| Mallik and Ray (2004) Exact | 0.787 | - |

Several parametric studies for post buckling behaviour are presented for undamaged and damaged composite plates in thermal environment after the various comparison studies for undamaged as well as damaged composite plates. The post buckling analysis is conducted for laminated composite plates (dimensions as in previous example) with PFCPs on the top surface (P/0/90/90/0). Initially, four PFCPs are placed as shown in Fig. 1. Material properties used for PFCs were $E$ = 63 GPa, $\nu$ = 0.31, $\rho$ = 7600 $kgm^{-3}$, $K_{11} = K_{22}$ = $15.3\times10^{-9}$ $Fm^{-1}$, $K_{33}$ = $15\times10^{-9}$ $Fm^{-1}$, $e_{11}$ = 14.14 $C/m^2$, $e_{21}$ = -3.34 $C/m^2$, $e_{24}$ = 10.79 $C/m^2$. The thermal expansion coefficients used were $\alpha_1 = 2x10^{-6}$ $^oC^{-1}$, $\alpha_2/\alpha_1$ = 10. Here also we have used a mesh size of 10 × 10 to discretize the whole plate and the center 4 elements (shown using grid in Fig. 1) are considered to have a mild damage. Post buckling behavior of such a simply supported plate is presented in Fig. 2. Further the post buckling behavior is studied with varying the number of PFCPs.

It is noted that for same loading, larger displacements are noticed in damaged composite plates compared to undamaged plates under 0 V cases. As noticed from results, an efficient control over the post buckling response can be obtained by applying voltages to the PFCPs. These results are noticed because of the lowering of the overall stresses in the composite substrate. The last stages of post buckling responses tend to neutral equilibrium position quicker as voltages are applied. This may be described by the fact that lesser resistance forces at post buckling stages of damaged composite plate when voltage is applied to PFCPs.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |

Figure 1. Position of 4 piezo patches (black shaded) in a simply supported plate with central damage.

## 4. CONCLUSIONS

This work has been done for post buckling analyses of damaged composite plate equipped with PFCPs. The governing equations are solved by using finite element method considering an eight noded iso-parametric element and using inverse hyperbolic shear deformation theory. The present theory helps to analyze complex problems under less computational complexity. The results got are quite accurate and show excellent performance of the present formulation. Validation of the current analysis showed desired outputs. The concept of anisotropic damage is used to incorporate the effects of damage in composite plate. Post buckling analysis for a damaged composite plate equipped with PFCPs subjected to mechanical and thermal loads are studied. The results show that the buckling temperature as well as post buckling strength of the composite plate with damage when no voltage is applied on PFC actuators is lower than that of the plate without damage. The results reveal that, the effect of applied voltage is more noticeable for the composite plate with PFC actuators, compared to the results of the composite plate with zero voltage on PFCPs. By applying voltage on PFCPs, deformations of damaged composite plate got counteracted and failure due to thermal environment is prevented. Thus the post buckling strength of composite plates with internal flaw can be enhanced by proper placement of PFCPs. It can be concluded that the use of PFCPs is a very promising means of designing new smart material applications due to the better capability of the smart composite actuator when PFCPs are used instead of continuous piezoelectric layer.

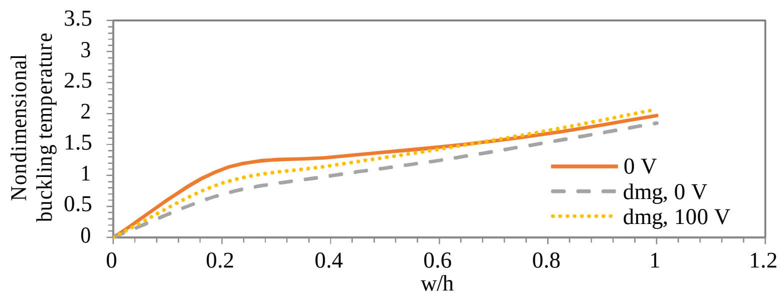


Figure 2. Thermal post buckling response for a simply supported plate with 4 PFCPs.

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## Biodata

Prof. Maiti did his B.E. degree in civil engineering in the year 1991 from Bengal Engineering College (currently IIEST), Shibpur under Calcutta University. Subsequently he was awarded M.Tech and PhD degrees from Department of Aerospace Engineering, IIT Kharagpur in the years 1993 and 1997 respectively. He is a national scholarship holder and MHRD scholar during 10+/B.E. and Master/Doctoral programs respectively. He was employed as Junior Project officer and Senior Research Engineer during August 1994 to December 1996 and December 1996 to May 1998 respectively. He subsequently moved to Aeronautical Development Agency, Bangalore as Scientist/Engineer and worked for over six and half years. He had joined department of Aerospace Engineering, IIT Kharagpur as a faculty member on October 2004 and presently he is holding the position of Professor and Head. He has published over 50 international journal papers, over 60 national and international conference papers, over 50 project reports, handled several research projects sponsored by ARDB, ADA, DST, ISRO, etc. of worth a few crores. So far 9 research students have obtained their PhD degree under his supervision. Currently 11 research students are pursuing their doctoral research work under his guidance. He has guided over 50 M.Tech students for their M.Tech Project works and over 50 B.Tech students for their B.Tech Projects. His primary research areas are analysis of composite structures under static and dynamic loadings employing various higher-order shear deformation theories, damage modelling of isotropic and composite materials, smart structures, aeroelasticity/aeroservoelasticity, structural health monitoring, etc.